SUPPLEMENTAL MATERIAL

of

ENHANCED OPPOSITE IMBERT–FEDOROV SHIFTS OF VORTEX BEAMS FOR PRECISE SENSING OF TEMPERATURE AND THICKNESS

By *Guiyuan Zhu, Binjie Gao, Linhua Ye, Junxiang Zhang, and Li-Gang Wang**
School of Physics, Zhejiang University, Hangzhou 310027, China
**lgwang@zju.edu.cn*

1. The DPS installation

The process of fabricating the air gap in our DPS is as follows. Two thin polyimide films with each thickness of 6 μm are sandwiched tightly between the left and right edges of these two prisms, creating an air gap between them under considerably large pressure by using the fastened screws. The DPS is firmly fixed to the rotation platform. A slight change in the rotation angle of the screw will affect the thickness of the air gap, so we can change the thickness of the air gap by fine-tuning the rotation angle of the screw. In our experiment, the thickness d of the air gap is measured by fitting the measured transmission curve (as a function of angle of incidence) of two different wavelengths of laser beams. Here, d is always smaller than the thickness of polyimide films due to the pressure from the fastened screws.

2. Determination of the air gap thickness by the double-wavelength method

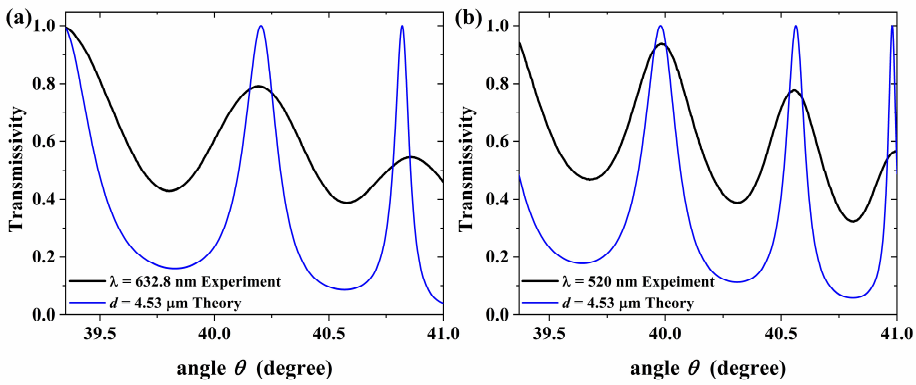

Fig. S1. Transmissivity of two beams with (a) $\lambda_1 = 632.8$ nm and (b) $\lambda_2 = 520$ nm. The black lines are the experimental results, and the blue lines are the theoretical simulations.

Figure S1 shows the transmissivity of two beams with λ_1=632.8 nm and λ_2=520 nm. It is seen that the two experimental results are in good agreement with the theoretical simulation. This is the double-wavelength method used to determine the thickness of the air gap. The thickness has a great influence on the angular interval of the transmittance peaks. The greater the thickness, the smaller the angular interval between the peaks. Therefore, d is determined by optimally fitting the peak interval of experimental transmissivity data.

3. The theoretical simulation of the IF shifts of vortex beams

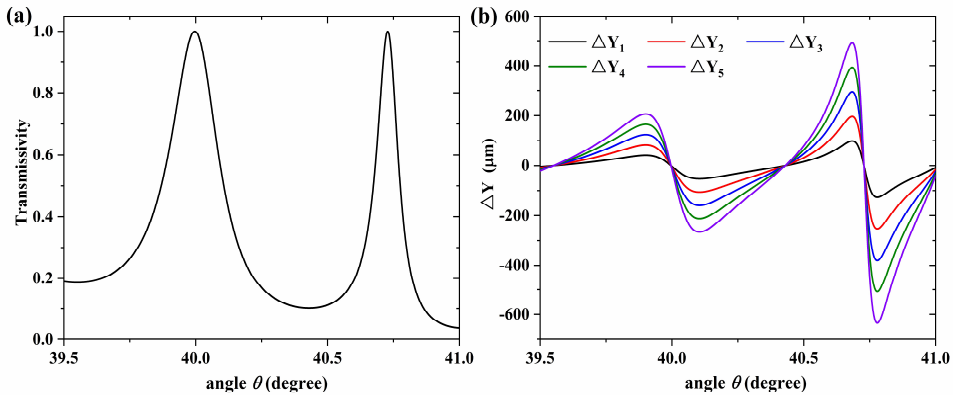

Fig. S2. The theoretical simulation of the transmissivity and relative IF shifts ΔY. (a) The transmissivity, (b) the black, red, blue, green, and purple curves denote the theoretical values of ΔY_1, ΔY_2, ΔY_3, ΔY_4, and ΔY_5, respectively. Here we take $d = 4.16$ μm.

According to Eqs. (1-2), we can get the theoretical results of IF shifts. Figure S2(a) shows the transmissivity of the DPS. Figure S2(b) shows the theoretical simulation of $\Delta Y_{|l|}$ with l = 1, 2, …, 5. The black, red, blue, green, and purple curves denote the theoretical relative shifts ΔY_1, ΔY_2, ΔY_3, ΔY_4, and ΔY_5, respectively. We can see that, as $|l|$ of opposite TCs increases from 1 to 5, the relative IF shift $\Delta Y_{|l|}$ increases. Thus, from the theoretical point of view, the OAM does have the effect on the IF shift. Meanwhile, from Fig. S2, we can also find that the relative difference $\Delta Y_{|l|}$ here becomes positive as the angle of incidence approaches the resonant angles and it becomes negative as the angle is away from the resonant angles.

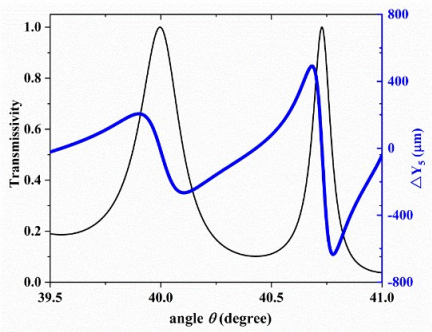

Fig. S3. The theoretical predictions of the transmission curve (the thin black curve, left axis) and the relative IF shift $\triangle Y_5$ (the thick blue curve, right axis). Here $\triangle Y_5$ is for the relative IF shift of vortex beams with ±5 TCs.

For the better explanations, we put the transmission curve and one of the relative IF shift together in Fig. S3. Now we can clearly find the relative IF shift becomes positive as the angle of incidence approaches the resonant angles (i.e., the value of θ is located at the left side of the resonant peak), and it becomes negative as the angle θ is away from the resonant angles (i.e., the value of θ is located at the right side of the resonant peak). According to Ref. [1] or Eq. (2), one can readily obtain the relative IF shift of vortex beams with opposite TCs as $\Delta Y_{|l|} \propto 2l\Theta_{GH}$, where Θ_{GH} is the angular GH shift of a fundamental Gaussian beam. As we know that for transmitted or reflected light, the angular GH shifts are proportional to the slopes of the transmissivity or reflectivity curves. Thus, when the transmission curve increases quickly with the increasing angle of incidence, the relative IF shift becomes the positive maximum, in contrast when the transmission curve decreases quickly with the increasing angle of incidence, the relative IF shift becomes the negative minimum. Thus, the change of the relative IF shifts between vortex beams with opposite TCs becomes maximal when the curve of transmissivity dramatically changes, and the relative IF shifts of vortex beams are zero when the angles of incidence are located at the resonant peaks or the valleys of the transmission curve. Thus, the change of IF shifts for vortex beams becomes maximal when the transmission curve changes quickly, whose underlying nature is mainly due to that the IF shifts here originate from the angular GH shifts [1].

4. The negligible angular IF shift for vortex beams with different TCs

According to the work in Ref. [1], the angular IF shift is given by $\Theta_{IF}^{l} = (1+2|l|)\Theta_{IF}$, where Θ_{IF} is the angular IF shift of a fundamental Gaussian beam. In our case, this angular IF shift for TE polarization is given by $\Theta_{IF} = -\frac{\theta_0^{\,2}}{2}\mathrm{Re}\left[2i\cot\theta\left(\frac{t_{TM}+t_{TE}}{t_{TE}}\right)\right]$ $= \frac{-\lambda^2}{2\pi w_0^{\,2}}\mathrm{Re}\left[2i\cot\theta\left(\frac{t_{TM}+t_{TE}}{t_{TE}}\right)\right]$, where θ_0 and w_0 are the angular spread and the waist of the incident Gaussian beam. This quantity Θ_{IF} is very tiny and it is less than 10^{-7} rad in our cases, which induces an additional displacement about 10^{-3} μm (much smaller than the spatial IF shifts of vortex beams). Here we provide the theoretical spatial and angular IF shifts of a Gaussian beam with the TE polarization in the below Fig. S4. From Fig. S4, we can see that

the value of the angular IF shift is very tiny (see Fig. S4(b)), and the spatial IF shift of the Gaussian beam is also much small (less than 1 μm), which is also much smaller than the spatial IF shifts of vortex beams with l=1, 2, and 3 (for examples, see Fig. S4(a)).

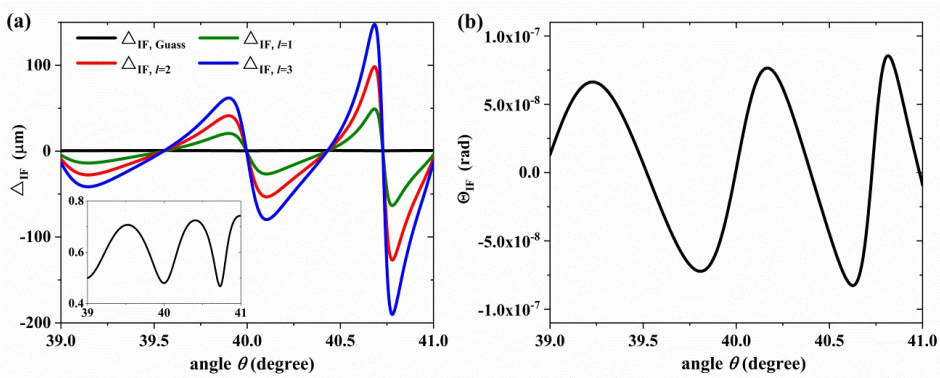

Fig. S4. (a) Comparison of the spatial IF shifts between a Gaussian beam (black) and vortex beams with l = 1 (green), 2 (red) and 3 (blue), and (b) the angular IF shift of a Gaussian beam in our double-prism system with the air thickness d = 4.16 μm. Here the beam parameter is w_o = 1 mm and the light beam is the TE polarization.

Furthermore, according to our proposal, we use the difference between the IF shifts of the vortex beams with opposite TCs as a probe of thickness and temperature sensing. Even if the displacements induced by the angular IF shifts of vortex beams with ±l TCs exist in our original data, they can be canceled since their displacements are the same in principle (only depending on the absolute value of TCs) as shown above.

5. Discussions of the theoretical and experimental amplification factors

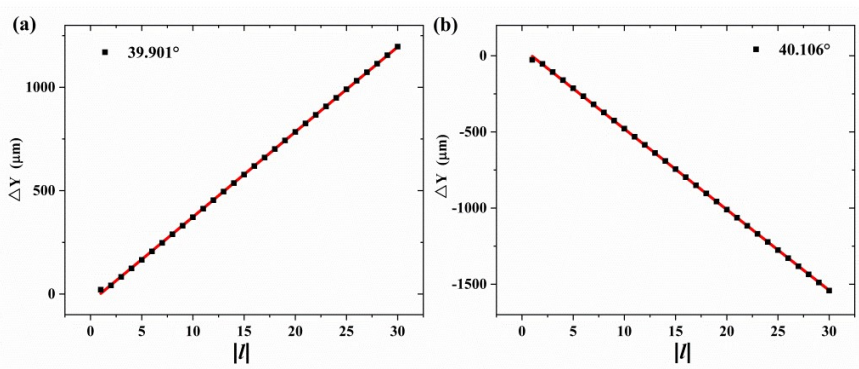

Fig. S5. Theoretical linear amplification relationship between ΔY and l. Their amplification factors (i.e., the slopes of the linear-fitted red lines) are (a) 41.137 μm/TC at θ = 39.901°, and (b) -52.991 μm/TC at θ = 40.106°, respectively.

According to Fig. S2 (b), we can obtain the theoretical amplification factors at different angles. Figure S5 shows the theoretical amplification factors (that is the slope of the linear-fitted lines) are 41.137 μm/TC for the maximum ΔY condition at θ = 39.901°, and -52.991 μm/TC for the minimum ΔY condition at θ = 40.106°, respectively. Note that this amplification factor is strongly dependent on the angles in the resonant structures since the relative IF shifts are oscillating with the angles of incidence as shown in Fig. S2(b). In principle, if the angle of incidence corresponds to the peak or valley of the transmission curve, i.e., when the slopes of the transmission curve vs the angle are zero, in these situations the amplification factors will be zero. When the angle of incidence corresponds to the dramatical change of the transmission curve, theoretically the maximal value of ΔY increases linearly as l increases. Thus, the sensitivity S of sensor can be enhanced by using larger topological charge.

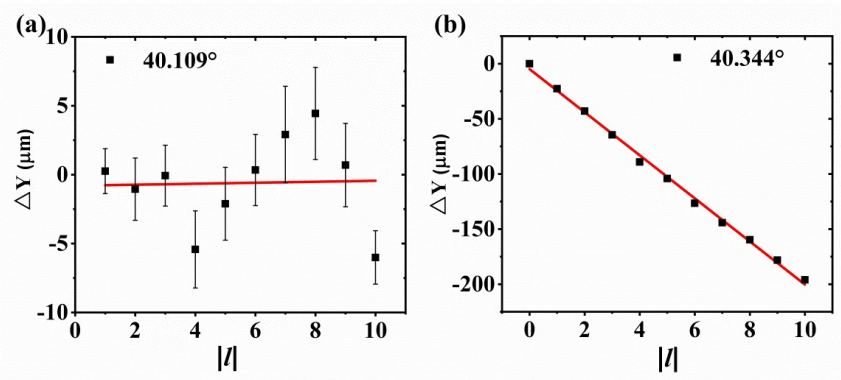

Fig. S6. Comparison of experimental amplification factors at two different angles: (a) the angle of practical resonance and (b) the angle where the transmission curve decreases quickly. The amplification factors (i.e., the slopes of the linear-fitted red lines) are 0.036μm/TC for the angle at θ = 40.109° (which is close to the resonant condition), and -19.535μm/TC for the angle at θ = 40.344° (where transmission curve drops quickly), respectively.

For the readers' reference, we provide the experimental data for the amplification factors in two limited cases. From Fig. S6, we can see that, near the angle of resonances the amplification factor (the slope of the linear-fitted red line) is almost zero (here it is 0.036 μm/TC), and the data are fluctuated around zero due to the experimental factors such temperature fluctuation and positioning accuracy of the angle scanning. When the angle is close to the situation that the transmission curve drops quickly, the amplification factor is -19.535 μm/TC in the case of Fig. S6(b). In our manuscript, we have provided the two situations, corresponding the quick increasing and decreasing of both sides of a transmission peak, respectively, see Figs. 3(c) and 3(d). Note that there exist differences between the theoretical predictions and experimental measurements on the angles since any slightly change in

temperature will lead to the thickness fluctuation of the air gap in the DPS, inducing the shifts of the practical resonant angles.

6. The experimental results of the IF shifts for TM and TE polarizations

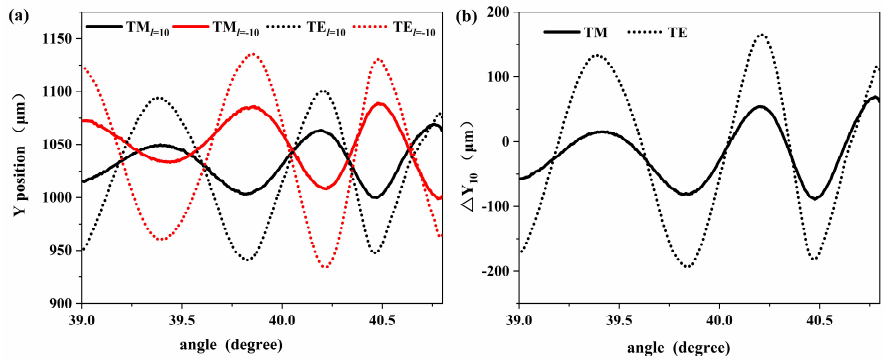

Fig. S7. The experimental results of Y position of the transmitted beam and relative IF shifts ΔY for TM and TE polarizations. (a) Y position of the transmitted beam for l = 10 and l = -10 for TE and TM polarizations, (b) the relative IF shifts ΔY_{10}, the black and red curves denote l = 10 and l = -10, respectively. The solid and dotted curves denote TM and TE polarizations, respectively, and here d = 4.55 μm.

Figure S7(a) shows the Y position of the transmitted beam for l = 10 and l = -10, and Fig. S7(b) shows their relative IF shifts ΔY_{10}, the black and red curves denote l = 10 and l = -10, respectively, and the solid and dotted curves denote TM and TE polarization, respectively. We can find that the change law for the IF shifts of vortex beams with opposite TCs for TM polarization is consistent with those for TE polarization, but the amplitude for TM cases was smaller. Thus, in our work we use the vortex beams with TE polarization. Note that the thickness d here is different from that in the manuscript and was confirmed from the above double-wavelength method. Before doing the experiment, we first confirmed the thickness of air gap in the DPS when the experiment was done in different days.

7. Comparison of the experimental results of the relative GH and IF shifts for vortex beams with opposite TCs

Figure S8 shows the comparison of the experimental data for the relative GH and IF shifts for vortex beams with opposite TCs. It is clear that the relative differences of GH shifts, ΔX_1,

ΔX_5 and ΔX_{10}, are almost independent of the value of TCs, and they are also much smaller than the relative differences of IF shifts, ΔY_1, ΔY_5 and ΔY_{10}, which are increased greatly as the value of TCs increases. Note that here the polarization of light is in the TE situation.

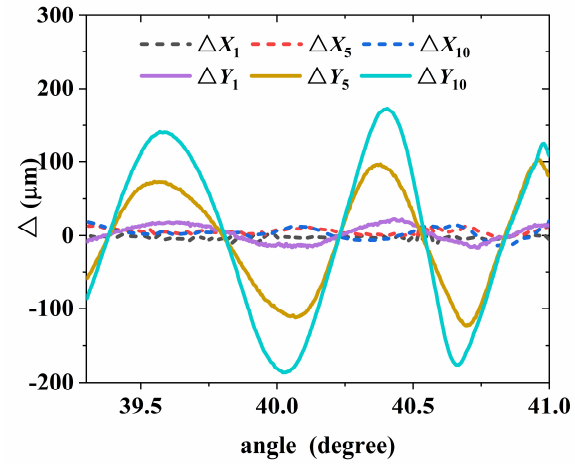

Fig. S8. The experimental results of the relative differences between GH shifts ΔX_1, ΔX_5 and ΔX_{10} and the relative differences between IF shifts ΔY_1, ΔY_5 and ΔY_{10}, for vortex beams with opposite TCs l=±1, ±5, ±10. Here d = 4.55 μm.

References

1. M. Merano, N. Hermosa, and J. P. Woerdman, "How orbital angular momentum affects beam shifts in optical reflection," Phys. Rev. A **82**, 023817 (2010).